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A multi-period location model with transportation economies-of-scale and perishable inventory

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ABSTRACT

In this paper, we propose a multi-period location model with transportation economies-of-scale that distributes a single perishable product. The problem involves a single supplier, a layer of potential facility locations, and a layer of retailers. Each facility is assumed to practice cross-docking and each retailer faces a constant demand rate in each planning period. A Zero-Inventory-Ordering (ZIO) inventory policy is assumed to be adopted at each retailer. The demand at each retailer must be satisfied by the end of the planning horizon, although backlogging is allowed in the intermediate periods. The decisions to be made comprise the location of facilities, the allocation of retailers to the open facilities with single-sourcing, and the logistics shipping plan over time for the open facilities and the retailers allocated to them. The goal is to minimize the total cost that includes (1) the fixed setup cost for locating facilities, (2) the inbound cost at the open facilities, (3) the transportation cost from the facilities to the retailers (which is assumed to be an economies-of-scale function), and (4) the inventory cost at the retailers. We first formulate the problem as a mixed integer nonlinear programming model. By effectively utilizing the structural properties of the cost functions and combining with the ZIO, we show how to linearize the nonlinear cost components. The results of a set of numerical experiments performed on randomly generated medium-sized problem instances are reported.

1. Introduction

The geographically dispersed suppliers, facilities, and retailers form the core logistical layers across a typical supply chain and an effective integrated network design is crucial to its success. Integrated logistics distribution network design is concerned with determining the logistics infrastructure (e.g., the location of facilities and the facility–retailer assignments) and the logistics planning operations such as inventory management and transportation. In this paper, we aim to develop a multi-period location–transportation network design model addressing the location, perishable inventory management, and transportation economies-of-scale issues to support such kind of decision-makings. We consider a single supplier that produces and distributes a single perishable product to the retailers over a finite number of planning periods. Each retailer reports its demand in each period to the supplier. Chan et al. (2002a, 2002b) and Shen et al. (2009) show that the Zero-Inventory-Ordering (ZIO) policy is very effective for a wide range of logistics planning problems. Thus, we assume that each retailer manages its inventory using a ZIO policy. The ZIO policy is a class of popular inventory replenishment policy for which the retailer places an order only when its inventory level drops to zero. The supplier is obligated to fulfill the demands of each retailer via a selected facility by the end of the planning period, although shortages and backlogging are allowed at the retailers during intermediate periods yet incur associated costs. To reduce the response time, provide good services, and achieve the transportation economies-of-scale, the supplier is assumed not to fulfill any demand via direct shipment and each retailer should source from a single facility. This is in line with the traditional uncapacitated facility location problem (UFLP) and many other location-inventory models such as Shen et al. (2003) and Teo and Shu (2004) in the literature. Instead, the supplier operates a number of uncapacitated facilities that practice cross-docking and coordinate the shipments between the supplier and the retailers. As shown by Simchi-Levi et al. (2003) and Ballou (2004), cross-docking is an effective logistics planning technique that aims at reducing logistics costs and improving service qualities by coordinating the operations of different logistical entities across

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a supply chain. Since each facility is assumed to adopt cross-docking, they thus do not hold any inventory. Like in Snyder et al. (2007), the location decision is assumed to be static that is made at the beginning of the planning horizon and does not change afterwards. The reason is that the location decision is primarily strategic which involves a substantial investment and therefore is hard to reverse. We also assume that the facility-retailer assignment decision is static, i.e., the allocation of the retailers to the facilities does not change over time. Its importance is highlighted in Shen and Max (2005) and Snyder et al. (2007) as static single-sourcing is adopted by many firms in their strategic network design. Static single-sourcing is also adopted by some other logistics distribution network models such as Freling et al. (2003), Huang et al. (2005) and Ahuja et al. (2007) in the literature. In contrast to the static network design decisions, the logistics shipping plan on the deliveries made to the open facilities and the retailers allocated to them changes over time. Clearly, the dynamic shipping plan has non-negligible impact on the transportation and inventory costs. Economies of scale is assumed for the transportation costs, which leads to complex non-convex cost functions. The objective is to provide the optimal solution so as to minimize the total cost that includes: (1) the fixed setup cost for locating the facilities, (2) the inbound cost at the facilities, (3) the transportation cost from the facilities to the retailers, and (4) the inventory cost at the retailers.

The rest of this paper is organized as follows. Section 2 reviews the recent research development on related problems. Section 3 presents two models for the multi-period location problem with transportation economies-of-scale and perishable inventory, namely the mixed integer nonlinear programming model and the linearized set-covering model. In Section 4, we present a greedy heuristic to solve the nonlinear discrete pricing problem so that the network design problem can be solved within the column generation framework. Computational results of a set of randomly generated instances are reported in Section 5. Finally, we conclude the paper in Section 6.

2. Literature review

One stream of research related to ours is the study of integrated location-inventory problems that aims at studying the impact of the operational level decision-makings of logistics on the strategic supply chain design decisions. Daskin et al. (2002) and Shen et al. (2003) propose a joint location-inventory problem that integrates the location, transportation, and warehouse inventory replenishment decisions. Gebennini et al. (2009), Miranda and Garrido (2009), Shen and Max (2005), Shu et al. (2005), Snyder et al. (2007), Sourirajan et al. (2007), Ozsen et al. (2008, 2009), and Park et al. (2010) study various important variants of the joint location-inventory problem. In contrast, Teo and Shu (2004), Üster et al. (2008), Keskin et al. (2010), Shu (2010), Shu et al. (2010), Keskin and Üster (2012), and Li et al. (2013) propose several location-inventory models that explicitly consider infinite horizon two-echelon inventory cost functions. Two-echelon inventory management needs to optimally coordinate the warehouse and the retailers to jointly replenish their inventories, which is still an open problem in the inventory literature. Due to the many different cost and service considerations in these models, these problems are usually very computationally challenging. By effectively exploiting the structural properties of these models, these papers propose column generation, Lagrangian relaxation, and other approximation approaches to address these problems. The aforementioned papers typically assume that the inventory is not perishable and the transportation cost function is linear. Lin et al. (2006) propose a location-inventory model that explicitly considers transportation economies-of-scale between the plants and the facilities. Each facility is assumed to adopt a one-for-one inventory replenishment policy to serve the retailers assigned to it. Inventories, which are not perishable, are only kept at the facilities and backlogging is not allowed. Furthermore, although the literature on location-inventory problems is relatively rich, the limited existing studies on multi-period location-inventory problems are contributed by Shen and Max (2005) and Snyder et al. (2007). Both models need to assume that the facility–retailer assignment vectors are period-dependent so that their models are tractable. The recent work of Soysal et al. (2015) incorporates environmental factors into the traditional two-echolen distribution system. Whereas the multi-period location-inventory model proposed in this paper complements the existing literature by considering the transportation economies-of-scale, the inventory perishability, and the facility–retailer assignment vectors being common across the entire planning horizon.

Our problem is also related to a class of multi-period single-sourcing problems in the literature. In a series of papers, Romeijn and Romero Morales (2001, 2003, 2004), Freling et al. (2003), Huang et al. (2005), and Ahuja et al. (2007) propose an important class of multi-period single-sourcing problems. They study the structural properties of these models; develop several efficient exact and greedy algorithms to solve these models; and use extensive computational tests to validate that their approaches can produce very high quality solutions efficiently. These models are related to our problem except that the location decision is ignored.

Another related stream of research is the perishable inventory management problems. Perishable inventory management has been studied within the logistics planning area (cf. Shen et al., 2009; Haijema, 2014). We refer to Nahmias (1982) for an excellent review on perishable inventory management.

3. Model formulation and the solution

In this section, we formulate the multi-period location problem with transportation economies-of-scale and perishable inventory that jointly optimizes the location, the facility–retailer assignment, and the inventory replenishment decisions. In this problem, we are given a single supplier, m potential locations for facilities, n retailers, and the length of T planning periods. The supplier produces and distributes a single perishable product to satisfy the demands of the geographically dispersed retailers. Facility specific fixed cost F_j is charged for opening and operating a warehouse at location j, $j = 1, \ldots, m$. Each retailer i faces a constant demand $D_{i,t}$ in the tth period, $i = 1, \ldots, n$; $t = 1, \ldots, T$. All demands must be satisfied by the end of the planning horizon. We assume that each retailer replenishes its inventory from an open facility using a ZIO policy for which the retailer places an order only when its inventory level drops to zero. We also assume that each facility is operated as a cross-docking facility, i.e., does not hold inventory, which cross-docks the inventory to each retailer it serves upon receiving the inventory from the supplier. Shortages and backlogging are allowed at the retailers in the intermediate periods. Each retailer i incurs a per unit holding cost rate $h_{i,t}$ for the amount of inventory held in period t and a per unit backordering cost rate $b_{i,t}$ for the amount of the demand backlogged in period t. The inbound shipment of consolidated orders from the supplier to each cross-docking facility incurs a linear transportation cost and the outbound shipment from each cross-docking facility to the retailers it serves incurs a non-convex piecewise linear transportation cost.

Since the inventory is perishable, we use $\alpha_{i,s,t} \le 1$ to denote the deterioration rate of inventory from time s to t at retailer i, i.e., one unit of inventory in time s shrinks to $\alpha_{i,s,t}$ unit in time t. Note that $\alpha_{i,s,t} = 1$ if $t \le s$, i.e., the inventory does not deteriorate if it is needed for backlog or used in the same period. Throughout this paper, we assume that the deterioration rate satisfies the condition

$$\alpha_{i,s,t} = \prod_{l=s}^{t-1} \alpha_{i,l}, \text{ for } s < t,$$

where $\alpha_{i,l}$ denotes the deterioration rate for each unit of inventory at retailer i and at time l (cf. Chu et al., 2005).

To facilitate the development of the model formulation, we let $P_{i,s,t}$ denote the amount of the demand of retailer i in time t that is satisfied by using the inventory received in time s. In addition, we use $Q_{j,s,i}$ to denote the amount of the inventory delivered to facility j in time s that is cross-docked directly to retailer i. We assume that whenever retailer i receives $Q_{j,s,i}$ units of the inventory from facility j at time s, it can use them to satisfy $P_{i,s,t}$ units of the demand at time t. In addition to determining the inventory replenishment decisions ($P_{i,s,t}$ and $Q_{j,s,i}$), we still need to simultaneously determine the location of facilities (denoted by y_j) and the assignment of retailers to facilities (denoted by x_{ij}). We let y_j be 1 if facility j is open and 0 otherwise. Also, we let x_{ij} be 1 if retailer i is assigned to facility j and 0 otherwise. We also let $X = \{x_{ij}: i = 1, \ldots, n; \quad j = 1, \ldots, m\}$. By defining the facility–retailer assignment vector X in this way, we implicitly assume that the facility–retailer assignment is identical across all planning periods, i.e., the facility–retailer assignment is period independent.

Before we formally present the mathematical formulation for the multi-period location problem with transportation economies-of-scale and perishable inventory, we first summarize the cost components and discuss their structural properties at the facility and the retailer level, respectively.

3.1. Cost components at facility level

At the facility level, we assume that for all $j = 1, \ldots, m$; $s = 1, \ldots, T$, it contains two costs: (1) Facility specific fixed cost F_j of opening and operating a facility at location j; (2) the inbound transportation cost incurred by facility j serving retailers in S at time s, which is assumed to be a linear function of the total shipping volume $\sum_{i\in S} Q_{j,s,i}$ received by facility j at time s. Let a_j denote the per unit shipment cost from the supplier to facility j. Then, the inbound transportation cost of facility j serving retailers in S at time s equals to $a_j \sum_{i\in S} Q_{j,s,i}$.

3.2. Cost components at retailer level

At the retailer level, we use $f_{i,j,s}(P_{i,s,1}, P_{i,s,2}, \ldots, P_{i,s,T}, X)$ to denote the aggregate transportation and inventory related costs of retailer i served by facility j in time s, which is a function of the replenishment plan $\{P_{i,s,t}: t = 1, \ldots, T\}$ and the facility–retailer assignment vector X. It depends on which facility serves retailer i and how retailer i utilizes the shipment received to fulfill the demand in each period. Hence, we need to define $f_{i,j,s}(\cdot)$ in terms of the amount $P_{i,s,t}$ used to satisfy the demand of retailer i at time t. The replenishment cost $f_{i,j,s}(\cdot)$ consists of two components: the outbound transportation cost denoted by $f^{Tr}_{i,j,s}(\cdot)$ and the inventory related cost denoted by $f^{In}_{i,s}(\cdot)$, both of which are functions of the replenishment plan $\{P_{i,s,t}: t = 1, \ldots, T\}$, i.e.,

$$f_{i,j,s}(P_{i,s,1}, \ldots, P_{i,s,T}, X) = f^{Tr}_{i,j,s}(P_{i,s,1}, \ldots, P_{i,s,T}, X) + f^{In}_{i,s}(P_{i,s,1}, \ldots, P_{i,s,T}).$$

In the following, we formulate these two cost components explicitly and discuss their structural properties.

- *Outbound Transportation Cost* $f^{Tr}_{i,j,s}(\cdot)$: Note that the total amount delivered from facility j to retailer i at time s is $Q_{j,s,i} = \sum_{t=1}^{T}(P_{i,s,t}/\alpha_{i,s,t}x_{ij})$. Then, the outbound transportation cost function becomes

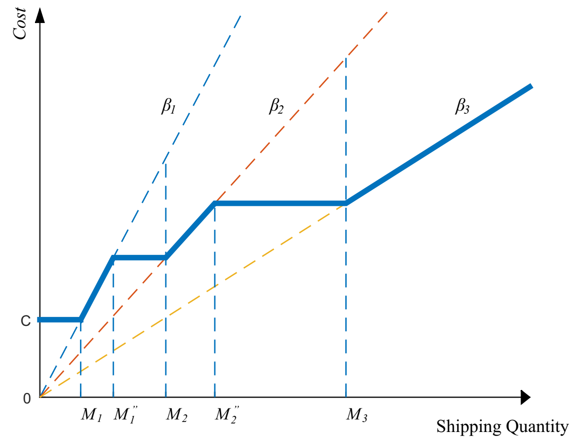

Fig. 1. Modified all unit discount cost structure.

$$f^{Tr}_{i,j,s}(P_{i,s,1}, \ldots, P_{i,s,T}, X) = f^{Tr}_{i,j,s}\Big(\sum_{t=1}^{T}\frac{P_{i,s,t}}{\alpha_{i,s,t}}x_{ij}\Big),$$

In practice, these shipments are usually delivered by Less-than-Truckload (LTL) carriers. Interestingly, in practice, when the shipper is going to ship L units, $M_i \le L < M_{i+1}$ and if $L \ge M_i'$, the shipping cost is charged as if they were shipping M_{i+1} units as shown in Fig. 1. This is called shipping L but declaring M_{i+1}. We use c to denote the minimum charge for shipping a small volume, i.e., c is the total cost when the shipping volume is no more than M_1. This commonly used practice implies that the outbound transportation cost function $f^{Tr}_{i,j,s}(\cdot)$ has the structure described by the solid line originated at point $(0, 0)$ as illustrated in Fig. 1. We refer to such cost functions as modified all-unit discount cost functions (cf. Chan et al., 2002a, 2002b). It is easy to see that the outbound transportation cost is non-increasing in each $P_{i,s,t}$.

- *Inventory Replenishment Cost* $f^{In}_{i,s}(\cdot)$: The inventory replenishment cost function $f^{In}_{i,s}(\cdot)$ can be formulated as

$$f^{In}_{i,s}(P_{i,s,1}, \ldots, P_{i,s,T}) = \sum_{l\ge s}\Big(\sum_{r=s}^{l} h_{i,r}\prod_{t=s}^{r-1}\alpha_{i,t}\Big)P_{i,s,l} + \sum_{l<s}(b_{i,l} + \ldots b_{i,s-1})P_{i,s,l},$$

where the first term corresponds to the total holding cost and the second is the total backordering cost. We note that this function term is linear and facility independent.

We assume that the demand incurred by each retailer can only be fulfilled by a single facility, i.e., single-sourcing is enforced. The objective is to minimize the total location, inbound and outbound shipments, and inventory holding and backordering costs. The general model for the multi-period location problem with transportation economies-of-scale and perishable inventory can be formulated as follows:

$$(P):\quad \min \sum_{j=1}^{m} F_j y_j + \sum_{j=1}^{m}\sum_{s=1}^{T} a_j\Big(\sum_{i=1}^{n} Q_{j,s,i}\Big) + \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{s=1}^{T} f_{i,j,s}(P_{i,s,1}, \ldots, P_{i,s,T}, X) \text{s. t.} \sum_{j=1}^{m} x_{ij} = 1, i = 1, \ldots, n \quad , \tag{1}$$

$$x_{ij} - y_j \le 0,\ i = 1, \ldots, n;\ j = 1, \ldots, m, \tag{2}$$

$$\sum_{s=1}^{T} P_{i,s,t} = D_{i,t},\ i = 1, \dots, n;\ t = 1, \dots, T, \tag{3}$$

$$Q_{j,s,i} = \sum_{t=1}^{T} \frac{P_{i,s,t}}{\alpha_{i,s,t}} x_{ij},\ i = 1, \dots, n;\ j = 1, \dots, m;\ s = 1, \dots, T, \tag{4}$$

$$x_{ij} \in \{0, 1\},\ i = 1, \dots, n;\ j = 1, \dots, m, \tag{5}$$

$$y_j \in \{0, 1\},\ j = 1, \dots, m, \tag{6}$$

$$P_{i,s,t} \ge 0,\ i = 1, \dots, n;\ s, t = 1, \dots, T. \tag{7}$$

The above formulation contains a nonlinear objective and a set of linear constraints with binary and continuous variables. Constraints (1) imply single-sourcing which ensures that each retailer is allocated to exact one open facility over the entire planning horizon. Constraints (2) require that each retailer can only be allocated to an open facility. Constraints (3) ensure that the demand of retailer i at time t must be satisfied by the inventory received across the entire planning horizon. Constraints (4) model if retailer i is served by facility j, the inventory cross-docked to retailer i from facility i at time s must be equal to the total demand across all time periods, after taking into account the deterioration rates. Constraints (5–7) are standard binary and non-negativity requirements on the variables. Thus, the problem (P) is a very complicated mixed integer nonlinear program that is almost impossible to be directly solved to optimality in an efficient way. We can observe that for a given assignment vector $\bar{X}$ and by letting S_j be the corresponding set of retailers assigned to facility j, the problem (P) can be decomposed into a set of $O(m)$ minimum-cost single-facility multi-retailer models $\min H(j, S_j)$, one for each facility j. The minimum-cost single-facility multi-retailer model (with facility j serving retailers in S_j) can be formulated as the following optimization problem:

$$\begin{aligned}
(H_j):\ & \min H(j, S_j) \\
\equiv \min\ & \sum_{s=1}^{T} a_j \left(\sum_{i \in S_j} Q_{j,s,i} \right) + \sum_{i \in S_j} \sum_{s=1}^{T} f_{i,j,s}(P_{i,s,1}, \dots, P_{i,s,T}, \bar{X}) \\
\text{s. t.}\ & \sum_{s=1}^{T} P_{i,s,t} = D_{i,t},\ \forall\, i \in S_j;\ t = 1, \dots, T, \\
& Q_{j,s,i} = \sum_{t=1}^{T} \frac{P_{i,s,t}}{\alpha_{i,s,t}},\ \forall\, i \in S_j;\ s = 1, \dots, T, \\
& P_{i,s,t} \ge 0,\ \forall\, i \in S_j;\ s, t = 1, \dots, T.
\end{aligned}$$

This subproblem model (H_j) is interesting in its own right since it takes several well-known operations management models studied in the literature as special cases. For example, it generalizes the model studied in Chu et al. (2005) and Chan et al. (2002a) to more general network topology. Unfortunately, (H_j) does not look tractable since it is still a non-convex optimization problem. The model (H_j) is also the same as the logistics planning model developed in Shen et al. (2009) except that the inbound transportation cost function is linear. Based on the linearization strategy adopted in Shen et al. (2009), in the rest of this section, we show how to linearize the nonlinear cost components in (H_j) by effectively utilizing the structural properties of the cost functions and combining this analysis with the ZIO policy.

We first show that (H_j) can be reformulated as a minimum cost multi-commodity network flow model by exploiting the properties of the ZIO policy. To define the network for the minimum cost multi-commodity network flow reformulation, let us consider retailer i in S_j. We define the graph $G_i(V_i, A_i)$ with vertices set V_i and arcs set A_i. Let $V_i := \{v_0, v_1, v_2, \dots, v_T\}$. For each pair of vertices $\{v_a, v_b\}$, there are T arcs linking them. Therefore, we have $A_i := \{e = (v_a, v_b, t):\ \ t = 1, \dots, T\}$. The replenishment plan of retailer i under a ZIO policy corresponds to a path from node v_0 to node v_T, i.e.,

$$\mathcal{P}_i = \{v_0 = \lambda_1, \lambda_2, \dots, \lambda_P = v_T\},$$

in $G_i(V_i, A_i)$, in which each arc $(\lambda_r, \lambda_{r+1}) = (v_a, v_b, s)$ represents the decision of using an order received in time s to satisfy all the demands incurred by retailer i from time period a to time period b, i.e.,

$$P_{i,s,a} = D_{i,a},\ P_{i,s,a+1} = D_{i,a+1},\ \dots,\ P_{i,s,b-1} = D_{i,b-1},\ P_{i,s,b} = D_{i,b},$$

with $P_{i,s,k} = 0$ for all $k < a$ and $k > b$. This is called the consecutive-cover-ordering property, i.e., the replenishment received by retailer i at time s is used to satisfy the demand over consecutive periods $[a, b]$, for some time period a and b. The cost associated with this arc is simply

$$c^i_{v_a, v_b, s} = f_{i,j,s}(P_{i,s,1}, \dots, P_{i,s,T}, \bar{X}).$$

By constructing the network in this way, it is easy to see that the following proposition holds.

Proposition 1. *The cost term* $\sum_{s=1}^{T} f_{i,j,s}(P_{i,s,1}, \dots, P_{i,s,T}, \bar{X})$ *for each* $i \in S_j$ *can be represented as a shortest path problem defined on* $G_i(V_i, A_i)$.

With Proposition 1 and noting that in this case the amount of the inventory cross-docked via facility j is

$$d^i_{v_a, v_b, s} = \sum_{t=1}^{T} \frac{P_{i,s,t}}{\alpha_{i,s,t}},$$

(H_j) can be reformulated as a minimum cost multi-commodity network flow formulation as follows:

$$\begin{aligned}
(MCMCNF):\ & \min_{w^i_{a,b,t}} LH(j, S_j) \\
\equiv \min\ & \sum_{t=1}^{T} a_j \left(\sum_{i, a<b} d^i_{a,b,t} w^i_{a,b,t} \right) \\
& + \sum_{i \in S_j} \left(\sum_{a<b,t} c^i_{a,b,t} w^i_{a,b,t} \right) \text{s. t.} \sum_{t,b:b>a} w^i_{a,b,t} - \sum_{t,b:b<a} w^i_{b,a,t} \\
= & \begin{cases} 1 & \text{if } (a = v_0), \\ -1 & \text{if } (a = v_T),\ \forall\, i \in S_j, \\ 0 & \text{otherwise}, \end{cases} \\
& 1 \ge w^i_{a,b,t} \ge 0,\ \forall\, a \le b,\ t,\ i.
\end{aligned}$$

Note that $w^i_{a,b,t} = 1$ if and only if retailer i uses the order received at time t to fulfill all the demands from time period a to b. The amount of the inventory cross-docked via facility j at time t, for retailer i, is thus $d^i_{a,b,t} w^i_{a,b,t}$. Therefore, the term $\sum_{i,a<b} d^i_{a,b,t} w^i_{a,b,t}$ refers to the total inventory transshipped through facility j at time t. All the function terms in $(MCMCNF)$ are now linear.

To this end, we have shown that how the subproblem (H_j) can be reformulated as a network flow model. In order to obtain a linear formulation for (P) and by the fact that the LP relaxation of the set-covering formulation can usually give a very tight bound, we thus reformulate (P) as a set-covering model (LP relaxation) as follows:

$$\min \left\{ \left(\sum_{j} \sum_{S} C_{j,S} \gamma_{j,S} \right) : \sum_{j} \sum_{S: i \in S} \gamma_{j,S} \ge 1,\ \forall\, i;\ 1 \ge \gamma_{j,S} \ge 0,\ \forall\, j \right\}, \tag{8}$$

where $C_{j,S} = F_j + \min_{w^i_{a,b,t}} LH(j, S)$ and $\gamma_{j,S}$ is 1 if facility j serves the maximal retailers set S and 0 otherwise. Based on the set-covering formulation (8), we will show how to apply a column generation approach to tackle (P) in the rest of this section.

We note that the number of columns in the set-covering formulation (8) is very huge. So, initially we can choose a subset of these columns with all singletons, which we call the master problem, to solve. If the reduced costs associated with those non-basic columns are non-negative, then we obtain the optimal solution to the set-covering formulation (8); otherwise we can find some columns with negative reduced costs, in case of which, we add these columns into the master problem and start the next iteration. The key is to have an efficient way to check the non-negativity of the reduced costs, which is equivalent to solve

$$\min_{w^i_{a,b,t}, j, S}\left(F_j + LH(j,S) - \sum_{i\in S}\pi_i\right), \tag{9}$$

where $\{\pi_i: i = 1, \ldots, n\}$ is the optimal dual solution obtained in one of the iterations of the column generation solution process. It is easy to see that checking the non-negativity of (9) is equivalent to determining whether there exists a solution (j,S) such that

$$\min_{w^i_{a,b,t}} LH'(j,S) \equiv \min_{w^i_{a,b,t}}\left(LH(j,S) - \sum_{i\in S}\pi_i\right) < -F_j. \tag{10}$$

As shown in the previous section, evaluating the function value of $\min_{w^i_{a,b,t}} LH'(j,S)$ for fixed j and S is an easy network flow optimization problem. However, the problem of $\min_{w^i_{a,b,t}, j, S} LH'(j,S)$ with j and S being variables is unlikely to be computationally tractable. Next, we propose an efficient greedy heuristic to address $\min_{w^i_{a,b,t}, j, S} LH'(j,S)$. Let I and J denote the set of retailers $i = 1, 2, \ldots, n$ and the set of potential facility locations $j = 1, 2, \ldots, m$, respectively. For each $j \in J$, we use the following greedy heuristic to solve $\min_{w^i_{a,b,t}, j, S} LH'(j,S)$.

Greedy Heuristic

STEP 0. Initially $S = \varnothing$ and set $\min_{w^i_{a,b,t}} LH'(j,\varnothing) := 0$.

STEP 1. Do while $S \neq I$ and $\min_{w^i_{a,b,t}} LH'(j,S) \geq -F_j$.

Choose $i \in I \backslash S$ such that

$$\min_{i\in I\backslash S}[LH'(j, S\cup\{i\}) - LH'(j,S)] \tag{11}$$

is minimized. Let i^* be the selected retailer that achieves the minimum value and set $S := S \cup \{i^*\}$. From the description of the greedy heuristic, it is not difficult to see it contains $O(n^2)$ steps.

In the next section, we report the computational results of using the column generation approach to solve the set-covering model (8).

4. Computational results

In this section, we summarize our computational experiences with the formulation and the solution approach outlined in the previous sections. We conducted the experiments on a dual-core CPU of 2 GHz and 2G RAM running the Windows 7 operating system. The reported CPU times exclude input times. All the cost parameters are randomly generated as follows:

- *Location and inventory related cost parameters*: F_j is generated uniformly in [600, 2000]; $\alpha_{i,s,t}$ is randomly generated in [0.5, 1]; $h_{i,t}$, $b_{i,t}$ are generated uniformly in [1, 2] and [2, 5], respectively; and $D_{i,t}$ is generated uniformly in [50, 200].
- *Inbound transportation cost parameters*: We randomly generate the per unit transportation cost from the supplier to facility j as follows: a_j is generated uniformly in [1, 3].
- *Outbound transportation cost parameters*: We randomly generate the modified all unit discount transportation cost structure for each retailer as follows: β_1 is generated uniformly in [6, 10] and M_1 in [1, 20]; β_2 is generated uniformly in $[3, \beta_1)$ and M_2 in (20, 40]; β_3 is generated uniformly in $[1.5, \beta_2)$ and M_3 in (40, 80]. Therefore, it is clear that $c = \beta_1 M_1$.

Table 1
Computational results with 6 periods: the impact of transportation economies-of-scale.

No. of locations	No. of retailers	No. of facilities open (1)	No. of facilities open (2)	CPU time MIP (seconds)	No. of columns generated
10	10	2	2	2.13	55
10	20	2	3	8.72	187
10	50	2	4	47.2	513
10	100	2	4	191.7	1638
20	10	2	2	3.85	101
20	20	2	3	19.71	426
20	50	3	4	101.2	1017
20	100	3	4	417.3	3321

Table 2
Computational results with 12 periods: the impact of transportation economies-of-scale.

No. of locations	No. of retailers	No. of facilities open (1)	No. of facilities open (2)	CPU time MIP (seconds)	No. of columns generated
10	10	1	3	2.83	51
10	20	2	3	12.2	181
10	50	2	4	56.6	536
10	100	2	4	221.2	1710
20	10	2	3	4.13	109
20	20	2	3	21.7	406
20	50	3	4	115.5	1036
20	100	3	5	435.3	3262

We note that the transportation cost structures generated above reflect the different quantity discount schemes. In reality, the inbound transportation is usually made by truckload carriers with a large shipping volume. Whereas, the outbound shipping volume is relatively smaller and usually made by less-than-truckload carriers. Due to this, the per unit cost associated with the inbound shipment is lower than the one under the outbound deliveries.

In order to examine the impact of transportation economies-of-scale on the design of a distribution network, we compare our model with a benchmark model without considering it. In the benchmark model, we consider a uniform per unit outbound transportation cost rate which is denoted by $\bar{\beta}$ and calculated as $\bar{\beta} = (\beta_1 + \beta_2 + \beta_3)/3$.

With these inputs, we solve the random problem instances using the CPLEX 11.0 LP solver. In case the LP solution to the final master problem is not integral, we apply a branch-and-price procedure to obtain the IP solution. Tables 1 and 2 present the computational results of the binary set-covering model and the comparison of it with the benchmark model with 6 periods and 12 periods, respectively. The input sizes of the tested random instances range from 6-period 10-location 10-retailer to 12-period 20-location 100-retailer. In both tables, the column titled "CPU time MIP (seconds)" reports the average CPU time needed to solve the binary IP formulation for different input sizes of the problem instances. The columns titled "No. of Facilities Open (1)" and "No. of Facilities Open (2)" give the average number of facilities open in implementing the model with transportation economies-of-scale and the benchmark model, respectively. We implement each class of the problem instances 20 times and report the average values of the number of facilities open (rounded to the nearest integer), CPU time, and the number of columns generated (rounded to the

Table 3
Computational results with 6 periods: the impact of perishable inventory.

No. of locations	No. of retailers	No. of facilities open (1)	No. of facilities open (2)	CPU time MIP (seconds)	No. of columns generated
10	10	2	2	1.74	43
10	20	2	2	7.27	168
10	50	2	2	51.3	527
10	100	3	2	205.2	1615
20	10	2	2	3.66	112
20	20	2	2	22.18	392
20	50	3	2	95.7	937
20	100	3	3	451.3	3073

Table 4
Computational results with 12 periods: the impact of perishable inventory.

No. of locations	No. of retailers	No. of facilities open (1)	No. of facilities open (2)	CPU time MIP (seconds)	No. of columns generated
10	10	2	1	2.75	47
10	20	2	2	14.6	173
10	50	3	2	67.3	525
10	100	3	2	278.8	1528
20	10	2	2	4.83	117
20	20	2	2	31.2	426
20	50	3	2	147.5	1153
20	100	3	2	572.4	3471

nearest integer). In most of the problem instances implemented, the LP relaxation gives us integral solutions. Tables 1 and 2 demonstrate that the medium-sized problem instances can be solved efficiently via our approach. For example, the average CPU time of solving the problem class with 12 periods, 20 potential locations, and 100 retailers is within 9 min. An important observation we obtain from the implementations is that the number of facilities open in the model with transportation economies-of-scale is consistently no more than it in the benchmark model and this phenomenon becomes more obvious when the number of planning periods increases. Unlike the traditional location-inventory models, all the facilities practice cross-docking and inventories are purely held by the retailers in our model. In most traditional studies, inventory cost consideration is reviewed as the most important reason for facility consolidation due to risk pooling. Weiskott (1998) shows that many companies are fond of streamlining and consolidating their distribution networks, especially for those electronics companies whose products are of high value and low weight. The inventory consideration, however, does not directly lead to a more consolidated distribution network in our case. In contrast, due to transportation economies-of-scale, our model encourages large volume shipment consolidation at the facilities and thus, opens fewer number of facilities.

We next show how the inventory deterioration rate affects the distribution network design. We implement and compare the models with perishable inventory and with nonperishable inventory (i.e., $\alpha_{i,t} = 1$ for all i, t). For the random instance with perishable inventory, we uniformly generated $\alpha_{i,t-1}$ in $(\alpha_{i,t}, 1)$, where $\alpha_{i,T}$ is uniformly generated in $[0.35, 0.5]$, i.e., the inventory deterioration rate is generated as a non-increasing sequence of the time period. All the other input parameters and the experiment setting are generated in the same way as in the previous experiments. The computational results are reported in Tables 3 and 4. The columns titled "No. of Facilities Open (1)" and "No. of Facilities Open (2)" give the average number of facilities open in implementing the model with perishable inventory and the model with nonperishable inventory, respectively. As in the previous experiments, the LP relaxation gives integral solutions for most test instances. Interestingly, we can observe that inventory deterioration does not encourage large volume shipment consolidation at the facilities, and thus need to open more facilities in the network design. This becomes more obvious when the number of planning periods increases. The inventory is replenished and used to satisfy the demand of each period in a more timely manner. In this situation, large volume delivery is not as preferred and consolidation is not an attractive option. Thus, the model with perishable inventory consistently uses more facilities in its distribution network than the model with nonperishable inventory.

5. Conclusions

In this paper, we propose a multi-period location model integrating facility location, perishable inventory replenishment, and non-convex economies-of-scale transportation cost functions. The model assumes that each facility practices cross-docking and functions as a transshipment point of inventory flows between the supplier and the retailers. Each retailer reports a constant demand rate to the supplier in each planning period. The inventory is perishable over periods and a retailer dependent deterioration rate parameter is used to measure this. We also assume that each retailer replenishes its inventory using a ZIO policy. Although shortages and backlogging are allowed in the intermediate periods at the retailers, all the demands are required to be satisfied by the end of the planning period. The inbound and outbound transportation costs are modeled as a linear and a piecewise linear non-decreasing economies-of-scale function, respectively. The objective is to minimize the total location, transportation, inventory holding, and backordering costs. We first formulate the problem as a nonlinear mixed integer programming model. By exploiting the structural properties of the economies-of-scale transportation cost function and the ZIO policy, we show how to linearize the model and reformulate it as a set-covering model. We propose a greedy heuristic to tackle the subproblem that must be solved in each iteration of the column generation procedure. We conduct a set of numerical experiments based on random problem instances with up to 12 periods, 20 location candidates, and 100 retailers. The computational results show that the proposed solution approach can efficiently solve the medium-sized problem instances. Through the computational study, we also examine how the economies-of-scale transportation and the inventory deterioration concerns affect the strategic network design, respectively.

The model proposed in this paper assumes that the demand at each retailer is known in each period. The model also assumes that the supply is always enough to match the demand by the end of the planning horizon. These ignored concerns limit the potentially practical application of the model. In reality, the demand and supply are often highly unpredictable for which stochastic and robust models are adopted to handle these uncertainties (cf. Shu and Song, 2014). Thus, it will be worthwhile to generalize the model to accommodate these important issues in future research.

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